EX 39 :

$\theta \in \left[0, \frac{\pi}{2}\right[$ $\qquad (E) : iz^2 + 6\sin\theta . z - 9i = 0$

1)

$\Delta' = 9\sin^2\theta + 9i^2 = -9 + 9.\sin^2\theta = -9(1 - \sin^2\theta) = -9\cos^2\theta \rightarrow \delta' = 3i\cos\theta$

a) $z' = \frac{-3\sin\theta + 3i\cos\theta}{i} = 3(\cos\theta + i\sin\theta)$ *et* $z'' = \frac{-3\sin\theta - 3i\cos\theta}{i} = 3(-\cos\theta + i\sin\theta)$

b) $z' = 3e^{i\theta}$ *et* $z'' = 3\left[\cos(\pi - \theta) + i\sin(\pi - \theta)\right] = 3e^{i(\pi - \theta)}$

2) $z_A = 3i$; $z_{M1} = 3(\cos\theta + i\sin\theta)$; $z_{M2} = 3(-\cos\theta + i\sin\theta)$

a) $|z_A| = |z_{M1}| = |z_{M2}| = 3 \Rightarrow OA = OM_1 = OM_2 = 3$ D'où $A, M_1, M_2 \in \zeta_{(0,3)}$

b) On a $OM_1 = OM_2$ donc OM_1AM_2 est un losange ssi :

$\overrightarrow{OM_1} = \overrightarrow{M_2A} \Leftrightarrow z_1 = z_A - z_2 \Leftrightarrow 3(\cos\theta + i\sin\theta) = 3i - 3(-\cos\theta + i\sin\theta)$

$\Leftrightarrow \cos\theta + i\sin\theta = i + \cos\theta - i\sin\theta \Leftrightarrow 2\sin\theta = 1 \Leftrightarrow \sin\theta = \frac{1}{2}$ *avec* $\theta \in \left[0, \frac{\pi}{2}\right[\Leftrightarrow \theta = \frac{\pi}{6}$

3) $i.z^4 + 3\sqrt{3}.z^2 - 9i = 0$ On pose $t = z^2$ l'équation devient :

$it^2 + 3\sqrt{3}t - 9i = 0 \rightarrow \Delta = -9 \rightarrow \delta = 3i$

$t' = \frac{-3\sqrt{3} + 3i}{2i} = 3\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ *et* $t'' = 3\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

• $t = 3\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \Leftrightarrow z^2 = 3e^{i\frac{\pi}{3}} \Leftrightarrow z = \sqrt{3}e^{i\frac{\pi}{6}}$ *ou* $z = \sqrt{3}e^{7i\frac{\pi}{6}}$

• $t = 3\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \Leftrightarrow z^2 = 3e^{2i\frac{\pi}{3}} \Leftrightarrow z = \sqrt{3}e^{i\frac{\pi}{3}}$ *ou* $z = \sqrt{3}e^{4i\frac{\pi}{3}}$

$S_C = \left\{\sqrt{3}e^{i\frac{\pi}{6}}; \sqrt{3}e^{7i\frac{\pi}{6}}; \sqrt{3}e^{i\frac{\pi}{3}}; \sqrt{3}e^{4i\frac{\pi}{3}}\right\}$

$N_1\left(\sqrt{3}e^{i\frac{\pi}{6}}\right)$; $N_2\left(\sqrt{3}e^{7i\frac{\pi}{6}}\right)$ $N_3\left(\sqrt{3}e^{i\frac{\pi}{3}}\right)$; $N_4\left(\sqrt{3}e^{4i\frac{\pi}{3}}\right)$